ESE

09.2.2b Hypotheses test of experiments with one factor and two treatments. Test Models. Examples.

Based on [K. Wohlin et al.]

t-test

Type	Parametric.
Assumptions	Two **independent** samples. Both of they come from a **normal** distribution.
Input	Samples (X:: X_1 ..X_n) and (Y:: Y_1 ..Y_m).
H_0	$\mu_X = \mu_Y$, i.e. the expected mean values are the same.
Calculation	Compute ($t_0 = t_X - t_Y$) $t_0 = (\bar{X}-\bar{Y}) / S_p \sqrt{((1/n)+(1/m))}$, $S_p \sqrt{(((n-1) S_X^2 + (m-1) S_Y^2) / (n+m-2))}$, S_X^2 and S_Y^2 are the individual sample variances.
Criterion	Reject H_o if $abs(t_0) > t_{\alpha/2,n+m-2}$, where the latter is the upper α percentage point of the t distribution with $n+m-2$ degrees of freedom (two sided test).

t-test example. Defect densities in different programs.

Input	X {3.42, 2.71, 2.84, 1.85, 3.22, 3.48, 2.68, 4.30, 2.49, 1.54} Y {3.44, 4.97, 4.76, 4.96, 4.10, 3.05, 4.09, 3.69, 4.21, 4.40, 3.49}
H_0	$\mu_X = \mu_Y$ $\alpha = 0.05$
Calculation	n=10, m=11, $\underline{X}$= 2.853, $\underline{Y}$=4,1055. S_X^2 = 0.6506, S_Y^2 = 0.4112, S_p= 0.7243, t_0= -3.96. f = 10+11-2=19, $t_{0.025,19}$ = 2.093
Conclusion	abs(t_0) > $t_{0.025,\ 19}$ → It is possible to reject the null hypothesis with a two tailed test at the 0.05 level.

Paired t-test

Type	Parametric.
Assumptions	Samples come in **pairs** from **repeated measures**. Both of them come from a **normal** distribution.
Input	Paired samples $(X_1, Y_1) \ldots (X_n, Y_n)$.
H_0	$\mu_d = 0$, i.e. the expected mean of the differences $d_i = x_i - y_i$ is 0.
Calculation	Compute $t_0 = \underline{d}\sqrt{n} / S_d$, where $S_d = \sqrt{(\Sigma_{i=1,n} (d_i - \underline{d})^2/(n-1))}$.
Criterion	Reject H_o if $abs(t_0) > t_{\alpha/2,n-1}$, where the latter is the upper α percentage point of the t distribution with n-1 degrees of freedom (two sided test).

Pair t-test example. Effort.

Input	Pm1 {105, 137, 124, 111, 151, 150, 168, 159, 104, 102} Pm2 { 86, 115, 175, 95, 174, 120, 153, 178, 71, 110}
H_0	$\mu_1 = \mu_2$ $\alpha = 0.05$
Calculation	d= {19, 22, -51, 16, -23, 30, 15, -19, 31, -9} S_d = 27.358, t_0= 0.39. n=10, f = 10 -1=9, $t_{0.025, 9} = 2.262$
Conclusion	$t_0 < t_{0.025, 19}$ → It is NOT possible to reject the null hypothesis with a two tailed test at the 0.05 level.

N.B. Non intervengono i campioni in quanto tali, ma solo la loro posizione nell'ordinamento totale di $X \cup Y$.

Mann-Whitney test

Type	Non-parametric. //An alternative to t-test
Assumptions	Two independent samples.
Input	Samples (X:: X_1 .. X_n) and (Y_1 ..Y_m).
H_0	Samples come from the same distribution.
Calculation	**Rank** all samples and compute both $U = N_{MIN}N_{MAX} + N_{MIN}(N_{MAX} + 1)/2 - T$, and $U' = N_{MIN}N_{MAX} - U$, where $N_{MIN} = \min(n, m)$, $N_{MAX} = \max(n, m)$, T is the Σ of the **ranks** of the smallest sample.
Criterion	Reject H_o if min(U, U`) is less equal to MannWhitneyTable (α) (N_{MIN}, N_{MAX}).

Mann-Whitney example. Defect densities in different programs.

Input	X {3.42, 2.71, 2.84, 1.85, 3.22, 3.48, 2.68, 4.30, 2.49, 1.54} Y {3.44, 4.97, 4.76, 4.96, 4.10, 3.05, 4.09, 3.69, 4.21, 4.40, 3.49}
H_0	Samples come from the same distribution. α= 0.05
Calculation	N_{MIN}= min(10, 11) =10, N_{MAX}= max(10, 11) = 11 Ranks(X) = {9, 5, 6, 2, 8, 11, 4, 17, 3, 1} Ranks(Y) = {10, 21, 19, 20, 15, 7, 14, 13, 16, 18, 12} T= 66, U= 99, U`=11 → MannWhitneyTable(0,05)(10, 11) = 26
Conclusion	min(U, U`) < tab(α, N_A, N_B) → It is possible to reject the null hypothesis with a two tailed test at the 0.05 level.

F test

Type	Parametric.
Assumptions	Two independent samples.
Input	Samples $(X_1 .. X_n)$ and $(Y_1 .. Y_m)$.
H_0	$\sigma^2_X = \sigma^2_Y$, i.e. the expected variances are equal.
Calculation	Compute $F_0 = \max(S_X^2, S_Y^2) / \min(S_X^2, S_Y^2)$, where S_X^2 and S_Y^2 are the individual sample variances.
Criterion	Reject H_o if $F_0 > F_{0, \alpha/2, CMAXV-1, CMINV-1}$, where C_{MAXV} is the number of items ("scores") in the sample set with maximum sample-variance, C_{MINV} is the number of scores with minimum sample-variance, $F_{\alpha/2, CMAXV-1, CMINV-1}$ is the upper α percentage point of the F distribution with CMAXV-1, CMINV-1 degrees of freedom (two sided test).

F test example. Defect densities in different programs.

Input	X {3.42, 2.71, 2.84, 1.85, 3.22, 3.48, 2.68, 4.30, 2.49, 1.54} Y {3.44, 4.97, 4.76, 4.96, 4.10, 3.05, 4.09, 3.69, 4.21, 4.40, 3.49}
H_0	$\sigma^2_x = \sigma^2_y$, i.e. expected variances <are equal, $\alpha = 0.05$
Calculation	$S_X^2 = 0.6506$, $S_Y^2 = 0.4112$, $F_0 = 1{,}58$. $C_{MAXV} = 10$, $C_{MINV} = 11$ $F_{0.025,\ 9,\ 10} = 3.78$
Conclusion	$F_0 < F_{0.025,\ 9,\ 10}$ → It is NOT possible to reject the null hypothesis with a two tailed test at the 0.05 level.