ESE

7.1 Experiment Planning. Experiment Design. Cases.

REPLICATION, RANDOMIZATION, BALANCING, AND BLOCKING IN EXPERIMENTAL DESIGN.

Some details

Additionally let us EXPERIECE WITH **ONE SOURCE OF DESIRED VARIATION**

O.1

One factor

One factor

Two treatments

O.1.1

MEMO: Experiment Design Types

One factor & (n>=2 treatments)

- **Randomized [Incomplete Block] Design**

  // A subject uses one treatment *(*)*

  (*) Subjects are assigned randomly to treatments

  - Simple *Randomized [IB] Design, SR[IB]D*

    // One treatment is applied to an object *(***)*

    (***) Objects are assigned randomly to treatments

  - Completely *Randomized [IB] Design, C[IB]RD*

    // All treatments are applied to an object *(****)*

- **Randomized Complete Block Design**

  // Each subject uses all the treatments *(**)*

  (**) The total order in which a subject uses the treatments is assigned at random.

  - [Simple] *RCBD* // One treatment is applied to an object *(***)*
  - [Completely] *RCBD* //All treatments are applied to an object *(****)*

    (****) The total order in which a treatment is applied to an object is assigned at random.

Subjects vs. Treatments

Treatments vs. Objects

Randomized [Incomplete Block] Designs

→ *One factor with two alternatives*

Whatever the level of
replication might be,
any participating subject
uses one treatment:
O.1.1.1

Completely Randomized Design

In order to begin, let us consider that all subjects work on a common object: **O.1.1.1.1**

Randomized [Incomplete Block] Design.
Completely Randomized [IB] Design

→ *One factor with two alternatives*

CR[IB]D: All treatments are applied to each object

For one factor, two treatments:

Basic experiment O.1.1.1 .1

Two subjects are involved, each working the **same object**,

&& each using a different treatment.

$o := O_1$; (o,S1, T1), (o,S2, T2)

The mapping among subjects and treatments is randomized.

Randomized [Incomplete Block] design.
Completely Randomized [IB] Design with Replications

→ *One factor with two alternatives*

All the experiment subjects are using the **same** object, && each subject will be using just one treatment.

This is a kind of boundary case.

Subjects will be randomly assigned to the treatments under study. If we have the same number of subjects per treatment, the study is balanced.

o=O_1 (o,S1, T1), (o,S2, T1), (o,S3, T2), (o,S4, T1) … (o,S10, T2)

The mapping among subjects and treatments is randomized and is possibly balanced.

Randomized [incomplete Block] design.
Completely Randomized [Incomplete Block] Design (with replications)

→ *One factor with two alternatives*

2 treatments, 2+8 subjects, 1 object

O

S	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}
A	T1	T1	T2	T2	T1	T2	T2	T2	T1	T1

Simple Randomized [Incomplete Block] Design

→ *One factor with two alternatives*

O.1.1.1 .2

Multiple objects are used,
but one treatment is applied to an object,
and object is worked by one subject,
whatever the level of replication might be.

Simple Randomized [Incomplete Block] Design

→ *One factor with two alternatives*

Basic experiment O.1.1.1 .2

1 object per treatment, and 1 subject per object:

(O1, T1, S1), (O2, T2, S2)

In the basic experiment, it is empty the intersection between any pair of elementary experiments

The mapping Treatments-Objects, and the further mapping with Subjects are both randomized (1 factor 2 alternatives)

O, S	O1, S1	O2, S2
T	T1	T2

Randomized [incomplete block] design.

Simple Randomized [IB] Design with …

Replication

→ *One factor with two alternatives*

One treatment per object, one subject per object:

$S_{i=1,2..n>}$ (O1, T1), (O3, T2), (O2, T2), (O4, T2) … (O10, T1)

The mapping 1 to 1 of objects and subject is randomized. The mapping of objects and treatments is randomized and hopefully balanced *(1 factor 2 treatments)*

O / S	O1 s1	O2 s2	O3 s10	O4 s9	O5 s4	O6 s8	O7 s3	O8 s5	O9 s6	O_{10} s7
A	T1	T2	T2	T2	T1	T2	T1	T2	T1	T1

Subjects (S) do change when passing from an object to another one; each subject uses just one object (O) and one treatment (T).

Completely Randomized **Complete** Block Design

→ One factor with two alternatives

All treatments are applied to each object.

O.1.1.2

Any subject uses all the treatments.

The advantage of this design technique is to see, for each assigned objects, the difference in performing all the treatments involved by the same subject.
However, in order to apply this approach, the learning effect must be low.

Randomized Complete Block Design.
Pair Comparison with replications

→ *One factor with two alternatives*

O.1.1.2.1
(S1, O1, T1), (S2, O1, T2), (S3, O1, T1), (S4, O1, T1) …
(S1, O1, T2), (S2, O1, T1), (S3, O1, T2), (S4, O1, T2) …

O.1.1.2.2
O.1.1.2.1 + …. (S1, O2, T1), (S2, O2, T1), (S3, O2, T2), (S4, O2, T1) …
(S1, O2, T2), (S2, O2, T2), (S3, O2, T1), (S4, O2, T2) …
… O3

O	O_k	
$T(S_i)$	T1	T2

Randomized Complete Block Design.
Pair Comparison with replications

→ One factor with two alternatives

(S1, O1, T1), (S1, O1, T2), (S2, O2, T2), (S2, O2, T1) … (S5, O5, T2)

In order to avoid the influence of which treatment should be started with first, we assign treatments to (a half of the) subjects in a random way.

O	O_1		O_3	O_4	
$T(S_1)$	T_1T_2	SPARE	T_2T_1	T_2T_1	SPARE
$T(S_2)$	T_2T_1		T_1T_2	T_1T_2	

One factor
More than two treatments

Completely Randomized [Incomplete Block] Design

→ *One factor with more than two alternatives*

CR[IB]D: All treatments are applied to each object, one treatment per subject

Completely randomized [IB] design with 3 treatments, (3+6) subjects, 1object

O_1	S	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9
	T	T2	T3	T1	T1	T2	T3	T3	T2	T1

Each treatment is applied 1+2 times to each object.

Completely Randomized Complete Block Design

→ One factor with more than two alternatives

CRCBD: All treatments are applied to each object, all treatments per subject.

CRCBD with 3 treatments, (3+3) subjects, 1object

O_1

S	S_1	S_2	S_3	S_4	S_5	S_6
1st T	T1	T3	T2	T2	T3	T1
2nd T	T3	T1	T3	T1	T2	T2
3rd T	T2	T2	T1	T3	T1	T3

Each treatment is applied 3+3 times to each the object

Multiple Design-Factors

EXPERIENCING WITH MULTIPLE SOURCES OF DESIRED VARIATION

Link to cardinalities of the elementary experiments

FACTORIAL DESIGN

K factors each with

2 Treatments

K=2 → 2x2 FACTORIAL DESIGN

Both factors have two treatments.

All the combinations of the treatments are possible.

F2 \ F1	a	b
A	S_4	S_1
B	S_2	S_3

K=3 → 2^3 FACTORIAL DESIGN

All the combinations of the treatments are possible.

F1	F2	F3	S
a	A	α	S2
b	A	α	S1
a	B	α	S8
b	B	α	S6
a	A	β	S5
b	A	β	S3
a	B	β	S7
b	B	β	S4

NESTED DESIGN

What to do when factorial treatments are not compatible

<table>
<tr><td colspan="4">Factor A</td></tr>
<tr><td colspan="2">Treatment A1</td><td colspan="2">Treatment A2</td></tr>
<tr><td colspan="2">Factor B</td><td colspan="2">Factor B</td></tr>
<tr><td>Treatment B1.A1</td><td>Treatment B2.A1</td><td>Treatment B1.A2</td><td>Treatment B2.A2</td></tr>
<tr><td>S_1</td><td>S_2</td><td>S_3</td><td>S_4</td></tr>
</table>

ONE HALF OF 2^3 FACTORIAL DESIGN

How to reduce the number of experiments? How to combine treatments of different factors?

?

F1	F2	F3	S
b	A	α	S8
a	B	α	S6
a	A	~~α~~ β	S5
b	B	~~α~~ β	S4

2^2 Factorial design

BLOCK DESIGN

Avoiding variations of no interest.

Have the uninteresting factor to influence equally all treatments.

BLOCK DESIGN

Block design with a single undesired variable

Apply each design factor treatment with each alternative of the blocking variable

BLOCK DESIGN

2 alternatives for
the undesired factor

1 interesting factor (I) 2 treatments (T1, T2)

Interesting factor (I)

Undesired factor (U)

U_{A1}	I_{T1}	I_{T2}
U_{A2}	I_{T2}	I_{T1}

In order to have the uninteresting factor to influence equally all treatments, 4 experiments are needed at least, i.e., 2 times the size of the factorial experiment with 1 interesting factor and 2 levels (1x2).

BLOCK DESIGN

2 alternatives for
the undesired factor

1 interesting factor (I) 3 treatments (T1, T2)

Interesting factor (I)

Undesired factor (U)

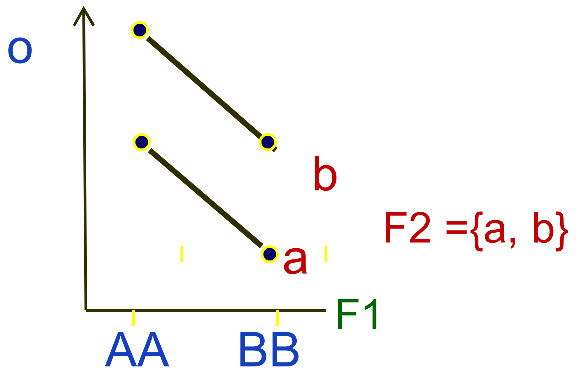
U_{A1}	I T1	I T2	I T3
U_{A2}	I T3	I T2	I T1

6 experiments at least.
Apply at random each design factor treatment with each alternative of the blocking variable.

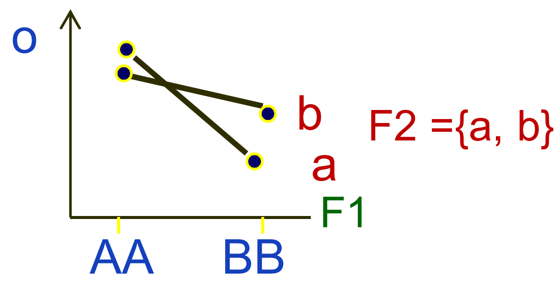
MULTIPLE DESIGN-FACTORS: INTERACTION

Hypotheses should consider

- not only the effect of a treatment on the output (o),
- but the effect $o_{f,t}$ of each treatment $F_f(t)$ of factor F_f on the output (o), and also
- the effect, if any, of interaction between $o_{f,t}$ and $o_{g,z}$



Not interacting factors
Best result: (AA, b), with ascending Y scale



Interacting factors
Best result: (AA, a), with ascending Y scale

Multiple Factors

Experiencing with multiple sources of undesired variations
(Continue) Link