UNIVERSITA' DEGLI STUDI DI ROMA "TOR VERGATA"
Dipartimento di Informatica, Sistemi e Produzione

8. REPLICATION, RANDOMIZATION, BALANCING, AND BLOCKING IN EXPERIMENTAL DESIGN

Some details

EXPERIECING WITH ONE SOURCE OF DESIRED VARIATION

O.1
One factor

One factor
O.1.1
Two treatments

Randomized [Incomplete Block] Designs

O.1.1.1
Any participating
subject
uses
one treatment,
whatever the level of replication might be.

→ One factor with two alternatives

Randomized [incomplete block] design.

Completely Randomized Design

O.1.1.1 .1

Additionally,

all subjects

work on a

common objects.

Randomized [incomplete block] design.
Completely Randomized Design

For one factor, two treatments:

Basic experiment O.1.1.1 .1

Two subjects are involved, each
working the **same object**,
&& using a **different treatment**.

$o = O_1$; (o,S1, T1), (o,S2, T2)

The mapping of subjects and treatments is randomized

Randomized [incomplete block] design.
Completely Randomized Design

Experiment with replications

All the experiment subjects are using the **same object**,
&& **each** subject will be using **just one treatment**.

This is a kind of boundary case too.

Subjects will be randomly assigned to the treatments under study. If we have the same number of subjects per treatment, the study is balanced.

$o=O_1$; (o,S1, T1), (o,S2, T1), (o,S3, T2), (o,S4, T1) … (o,S10, T1)

The mapping among subjects and treatments is randomized and is possibly balanced

Randomized [incomplete block] design.
Completely Randomized Design (with replications)

2 treatments, 2+8 subjects, 1 object

O										
S	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}
A	T1	T1	T2	T2	T1	T2	T2	T2	T1	T1

Randomized [incomplete block] design.

Simple Randomized Designs

O.1.1.1 .2

More then one **objects** are used, but
an object is worked by one subject only,
whatever the level of replication might be.

Randomized [incomplete block] design.
Simple Randomized Design

Basic experiment

1 object **per** treatment, and **1** subject **per** object:

(O1, T1, S1), (O2, T2, S2)

In the basic experiment, it is empty the intersection between any pair of elementary experiments

The mapping Treatments-Objects, and the further mapping with Subjects are both randomized
(1 factor 2 alternatives)

O, S	O1, S1	O2, S2
T	T1	T2

with …

Randomized [incomplete block] design.

Simple Randomized Design …. **Replications**

One treatment per some objects, **one** subject **per** object:

$S_{i=1,2..n>=2}$; (O1, T1), (O3, T2), (O2, T2), (O4, T2) … (O10, T1)

The mapping 1 to 1 of objects and subject is randomized. The mapping of objects and treatments is randomized and hopefully balanced *(1 factor 2 treatments)*

O / **S**	O1 **s1**	O2 **s2**	O3 **s10**	O4 **s9**	O5 **s4**	O6 **s8**	O7 **s3**	O8 **s5**	O9 **s6**	O_{10} **s7**
A	T1	T2	T2	T2	T1	T2	T1	T2	T1	T1

Subjects (S) do change when passing from an object to another one; each subject uses just one object (O) and one treatment (T).

Randomized Complete Block Design

O.1.1.2

Any subject uses all the treatments.

The advantage of this design technique is to see the difference in performing the treatments involved by the same subject.

However, learning effect is expected to be low.

Pair comparison

Pair comparison is the name given to Randomized Complete Block Design when there is one factor with two treatments.

O.1.1.2.1

Pair randomized design: Basic experiment

(S, O, T1), (S, O, T2)

O	O	
A(S)	T1	T2

Pair comparison with Replications

O.1.1.2.1

(S1, O1, T1), (S2, O1, T2), (S3, O1, T1), (S4, O1, T1) …
(S1, O1, T2), (S2, O1, T1), (S3, O1, T2), (S4, O1, T2) …

O.1.1.2.2

O.1.1.2.1 + **…. (S1, O2, T1), (S2, O2, T1), (S3, O2, T2), (S4, O2, T1) …**
(S1, O2, T2), (S2, O2, T2), (S3, O2, T1), (S4, O2, T2) …
… O3

O	O_k	
$A(S_i)$	T1	T2

Pair comparison
(with replications)

(S1, O1, T1), (S1, O1, T2), (S2, O2, T2), (S2, O2, T1) … (S5, O5, T2)

In order to avoid the influence of which treatment should be started with first, we assign treatments to (a half of the) subjects in a random way.

O	O_1	SPARE	O_3	O_4	SPARE
$T(S_1)$	T_1T_2		T_2T_1	T_2T_1	
$T(S_2)$	T_2T_1		T_1T_2	T_1T_2	

One factor
More than two treatments

Completely randomized design

Completely randomized design with 3 treatments, (3+7) subjects, 1object

	S	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}
O_1	T	T2	T3	T1	T1	T2	T3	T3	T2	T1	T2

→ One factor with more than tewo alternatives

Randomized complete block design

3 treatments, (3+3) subjects, 1object

O_1

S	S_1	S_2	S_3	S_4	S_5	S_6
1st T	T1	T3	T2	T2	T3	T1
2nd T	T3	T1	T3	T1	T2	T2
3rd T	T2	T2	T1	T3	T1	T3

EXPERIENCING WITH MULTIPLE SOURCES OF DESIRED VARIATION: (Multiple Design-Factors)

→ 24

Multiple Design-Factors

Hypotheses should consider not only the effect $\varepsilon_{i,j}$ of each treatment $F_i(j)$ on factor F_i, but also the effect of interaction between $\varepsilon_{i,j}$ and $\varepsilon_{e,f}$

DESIGN WITH ONE VARIATION AT A TIME

Number of experiments, N, when the best out of n_i treatments of the i^{th} factor is selected for evaluating the $(i+1)^{th}$ factor.

$$N=1+\Sigma_{i=1,.K}\,(n_i-1)$$

FACTORIAL DESIGN

STUDYING INTERACTIONS

Number of experiments with K factors, the i^{th} with n_i treatments

$$N = \prod_{i=1,.K} n_i$$

FACTORIAL DESIGN

K factors each with

2 Treatments

2x2 FACTORIAL DESIGN

Both factors have two treatments, and every combinations of the treatments are possible

F2 \ F1	a	b
A	S_4	S_1
B	S_2	S_3

2^3 FACTORIAL DESIGN

F1	F2	F3	S
a	A	α	S2
b	A	α	S1
a	B	α	S8
b	B	α	S6
a	A	β	S5
b	A	β	S3
a	B	β	S7
b	B	β	S4

NESTED DESIGN
What to do when factorial treatments are not comparable

Factor A			
Treatment A1		Treatment A2	
Factor B		Factor B	
Treatment **B1.A1**	Treatment **B2.A1**	Treatment **B1.A2**	Treatment **B2.A2**
S_1	S_2	S_3	S_4

2^3 FACTORIAL DESIGN

How to reduce the number of experiments

One-half of fractional design of 2^3 FACTORIAL DESIGN

F1	F2	F3	S
b	A	α	S8
a	B	α	S6
a	A	αβ	S5
b	B	αβ	S4

2^2 Factorial design

Avoiding variations of no interest:

BLOCK DESIGN

**Have the uninteresting factor
to influence equally all treatments.**

COMPLETE BLOCK DESIGN

Apply each design factor treatment with each alternative of the undesired variables

Block design with a single undesired variable

Apply each design factor treatment with each alternative of the blocking variable

2 alternatives for the undesired factor

1 interesting factor (I), 2 treatments (T1, T2)

Interesting factor (I)

Undesired factor (U)

U_{A1}	I_{T1}	I_{T2}
U_{A2}	I_{T2}	I_{T1}

In order to have the uninteresting factor to influence equally all treatments, 4 experiments are needed at least, i.e. 2 times the size of the factorial experiment with 1 interesting factor and 2 levels (1x2).

2 alternatives for the undesired factor

1 interesting factor, 3 treatments

Interesting factor

Undesired factor

U_{A1}	I_{T1}	I_{T2}	I_{T3}
U_{A2}	I_{T3}	I_{T2}	I_{T1}

6 experiments at least

Apply at random each design factor treatment with each alternative of the blocking variable

Apply at random each design factor treatment with each alternative of the blocking variable

4 alternatives for the undesired factor

e.g. Team (experience)

1 interesting factor, 4 treatments (A, B, C, D)

Block size

U_{A1}	C	B	A	D
U_{A2}	A	B	D	C
U_{A3}	B	C	D	A
U_{A4}	A	D	C	B

Block

16 experiments at least, i.e. 4 times the factorial experiments (1x4)

4 alternatives for the undesired factor (e.g. team exp. $T_{i=1..4}$)

1 interesting factor, 3 treatments (A, B, C)

U_{A1}	C	B	A
U_{A2}	A	B	C
U_{A3}	B	C	A
U_{A4}	A	C	B

12 experiments at least

Apply at random each design factor treatment with each alternative of the blocking variable

Reduce the complexity of the experiment but have the uninteresting factor to influence equally all treatments.

Block design with two sources of undesired variability

4 alternatives for each of the 2 undesired factors

1 interesting factor, 4 treatments (A, B, C, D)

U2 / U1	VS	S	L	VL
U1.1	A	B	C	D
U1.2	D	A	B	C
U1.3	C	D	A	B
U1.4	B	C	D	A

e.g. Project size

Each interesting treatment 1 time both per row and column

16 experiments at least

LATIN SQUARE

Block design with three or more sources of undesired variability

3 undesired factors; 4 alternatives for each of such factors

1 interesting factor, 4 treatments (α, β, χ, δ)

Each interesting treatment 1 time both per row and column

U1 \ U2	VS	S	L	VL
U1.1	Aα	Bβ	Cχ	Dδ
U1.2	Dδ	Aα	Bβ	Cχ
U1.3	Cχ	Dδ	Aα	Bβ
U1.4	Bβ	Cχ	Dδ	Aα

U3

U3: {A, B, C, D}

GRECO-LATIN SQUARE

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4 undesired factors; 4 alternatives for each of such factors

1 interesting factor, 4 treatments (α, β, χ, δ)

U2 / U1	VS	S	L	VL
T1	aAα	bBβ	cCχ	dDδ
T2	dDδ	aAα	bBβ	cCχ
T3	cCχ	dDδ	aAα	bBβ
T4	bBβ	cCχ	dDδ	aAα

?

U3U4

4 alternatives for each of the undesired # 4 factors

1 interesting factor, 4 treatments (α, β, χ, δ)

U1 \ U2	VS	S	L	VL
$U1_1$	aAα	bBβ	cCχ	dDδ
$U1_2$	bCδ	aDα	dAβ	cBχ
$U1_3$	cDχ	dCδ	aBα	bAβ
$U1_4$	bDβ	cAχ	bDδ	aCα

U3U4

?

4 alternatives for each of the undesired # 4 factors

1 interesting factor, 4 treatments (α, β, χ, δ)

Each treatment & each alternative 1 time both per row and column, and each & couple of alternatives 1 time

U1 \ U2	VS	S	L	VL
$U1_1$	$X_1Y_1\alpha$	$X_2Y_2\beta$	$X_3Y_3\chi$	$X_4Y_4\delta$
$U1_2$	$X_2Y_3\delta$	$X_1Y_4\alpha$	$X_4Y_1\beta$	$X_3Y_2\chi$
$U1_3$	$X_3Y_4\chi$	$X_4Y_3\delta$	$X_1Y_2\alpha$	$X_2Y_1\beta$
$U1_4$	$X_2Y_4\beta$	$X_3Y_1\chi$	$X_2Y_4\delta$	$X_1Y_3\alpha$

U3U4

HYPER-GRECO-LATIN SQUARE

Incomplete block design

4 treatments for the desired factor and 1 blocking variable

Block \ Factor	A	B	C	D
1	X	X	X	
2	X	X		X
3	X		X	X
4		X	X	X

4 blocks, 12 experiments: each pair of treatments occur the same number of times in the blocks.